A Markov Decision Evolutionary Game for the study of a Dynamic Hawk and Dove Problem

Eitan Altman, Ilaria Brunetti

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Abstract

In this paper we study one of the most well known examples of evolutionary games, the Hawk and Dove problem, in the dynamic framework of Markov Decision Evolutionary Games. We associate with each player an extra individual state depending on the age and on the strength of the individual. This state may change as a function of the actions taken by those it encounters. The goal of a player is to maximize the expected sum of its immediate fitness during its life time. We identify a unique stationary equilibrium in the game and compute its value. We further extend the Hawk and Dove game by introducing Group Markov Decision Evolutionary Game Theory (GMDEG), in which a player does not necessarily represent a single interacting individual but a whole class of such individuals. The fitness to be maximized is the one of the group: this approach shows novel results on the structure of the equilibria and the non uniqueness of the equilibria.

1 Introduction

One of the most studied examples in evolutionary games is that of Hawks and Doves [4, 5]. This model has been extensively studied for modeling, predicting and explaining the level of aggressiveness in a population in which there is a competition of individuals over natural resources. This game was also used in Engineering applications to study competition of congestion control algorithms [2, 1] in communication networks as well as to study the interactions between mobile phones that can choose which power to use when transmitting packets [6]. In this paper we revisit this standard game within the framework of Markov Decision Evolutionary Games (MDEG) [3], by introducing extra individual states, related to the age and the strength of the individual. By introducing these states we are able to evaluate the impact of adults' aggressive behavior on the evolution of young individuals within a population of players. We introduce a dynamic dimension to the problem by supposing that a young individual can evolve either to a strong adult or to a weak adult, depending on whether it was attacked when young. Only strong adults can be aggressive, whereas weak adults or young individuals have no choice of actions. This game differs from the classical one in several aspects. First, to each individual there is an associated state that may change as a function of the actions taken by those it encounters. Second, a player's objective is not to maximize its immediate fitness but to maximize the expected sum of its immediate fitness during his life time. We identify a unique stationary equilibrium in the game and compute its value.

We further extend the Hawk and Dove game by introducing Group Markov Decision Evolutionary Game Theory (GMDEG), in which the concept of the player as a single individual is replaced by that of a whole group of individuals. Even if we still consider pairwise interactions among individuals, our perspective is completely different: we suppose that individuals are simple actors of the game and that the utility to be maximized is the one of their group.

We found several examples which suggested the need of this new group approach:

- in some species like bees or ants, the one who interacts is not the one who reproduces. This implies that the Darwinian fitness is related to the entire swarm and not to a single bee.
- In many species, we find altruistic behaviors, which may hurt the individual adopting them, favoring instead the group it belongs to. Altruistic behaviors are typical of parents toward their children: they may incubate them, feed them or protect them from predators at a high cost for themselves. Another example can be found in flock of birds: when a bird sees a predator it gives an "alarm call" to warn the rest of the flock, attracting the predator's attention to itself. Also the stinging behavior of bees is an altruistic one: it serves to protect the hive, but it's lethal for the bees which strives.
- In engineering applications: in wireless communication, power control games have frequently been studied in the framework of standard EGT. Papers that consider these games always assume that each mobile can control selfishly its power. In practice however the protocols for power control are not determined by the users of the terminal but by the equipment constructors; this implies that the real competition is among a final number of equipment constructors.

  By applying this group approach to the Hawk and Dove MDEG, we find novel results on the structure of the equilibria: we show that in contrast to the classical Hawk and Dove game the pure non aggressive strategy can be an equilibria and also that there are some values of the parameters such that the game has two different equilibria, one in pure strategies and one in mixed ones.

  The structure of the paper is as follows. In the next section we provide a short overview on Markov Decision Evolutionary Games and we present an MDEG approach for the Hawk and Dove game. In section 3 we derive the expected fitness and the Equilibria for this game. In Section 4 we study the Hawk and Dove game as a GMDEG problem, we define the fitness of the game and we compute the equilibria.

2 MDEG

2.1 Preliminaries of MDEG

We begin by describing the game model which falls into the category of MDEG [3]. We consider an infinitely large population of players, in which pairwise interactions among individuals occur. Each player born at some random time t_0 and interacts with another randomly selected individual at each instant of time t. After a certain period the player dies and is replaced by another individual. We associate with each player a Markov Decision Process (MDP). The parameters of the MDP are given by tuple $\{\mathcal{S}, \mathcal{A}, \mathcal{Q}\}$, where:

- $\mathcal{S}$ is the set of possible individual states;

- $\mathcal{A}$ is the set of available actions. For each state $s \in \mathcal{S}$, $\mathcal{A}_s$ is the subset of available actions for a player in state s.
- $\mathcal{Q}$ is the set of transition probabilities. Given the states s, $s' \in \mathcal{S}$ of an individual, and the actions $a, a' \in \mathcal{A}$, $\mathcal{Q}_{s'}(s, a, a')$ is the probability to move from state s to state s' taking action a when interacting with an individual that takes action a'.

We shall assume that each individual restricts to stationary policies, denoted by $\mathcal{U}_{\mathcal{S}}$, i.e., policies that depend only on its current state. For each given initial state x of a player and each policy u of that player, which we call a tagged player, if the rest of the population uses some common stationary policy v, then the state and the action process of the tagged player form an MDP whose distribution depend on the initial state x and on the policies u and v. We define the fraction of players at each of the possible individual state as the global state. We assume that if the rest of the population uses some common stationary policy v, then the global state process before the tagged user is born forms a time-homogeneous Markov chain. We assume that when the tagged player is born, he finds that Markov chain in steady state.

We shall denote by $\mathbb{P}_{x,u,v}$ the probability measure induced by the initial state x and strategies u of the tagged player and the strategy v for the rest of the population, where we assume that upon the birth of the tagged player, it finds the system in steady state.

Let $r(s_t, a_t, s'_t, a'_t)$ be the immediate reward of a player in state s, playing action a against a player in state s' playing a' at discrete time t. We introduce the expected lifetime fitness of a tagged player whose lifetime goes from time 0 to time τ:

$$F_x(u, v) := E_{x,u,v}[\sum_{t=0}^{\tau} r(s_t, a_t, s'_t, a'_t)] \tag{1}$$

In the following section, we shall restrict to stationary policies.

2.2 An MDEG approach for the Hawk and Dove Game

We revisit one of the most studied examples in evolutionary games, that of the Hawk and the Dove, in this dynamic context. Our main goal is to study the impact of the aggressive behavior of adults on the evolution of young individuals. We define a four state model: every individual born young and after each pairwise interaction can become an adult or remain in young state. If a young meets an aggressive adult either it evolves as a weak adult or it becomes a weak young. A weak young, when evolving can only become a weak adult, whereas a young who has never been attacked can also evolve into a strong adult.

More precisely, the tuple $\{\mathcal{S}, \mathcal{A}, \mathcal{Q}\}$ describing our game is defined as follows:

- The set of states is $\mathcal{S} = \{Y, Y_W, A_S, A_W\}$, where Y, Y_W correspond to young and weak young, A_S to strong adult and A_W to weak adult.
- $\mathcal{A} = \mathcal{A}_{A_S} = \{H, D\}$: only strong adults can choose an action. The available actions in state A_S are Hawk (H) and Dove (D), which correspond respectively to play aggressively and not aggressively.
- We define the probability of remaining in the young state for a young individual as $q \in [0, 1]$; consequently, the probability for a young individual to evolve into an adult one is $1 - q$. Analogously, the probability for an adult to stay in adult state is $p \in [0, 1]$;

with probability $1-p$ an adult individual dies and it's replaced by another young individual. This allows us to describe the set of transition probabilities $\mathcal{Q}$:

- If a young individual meets an adult one, the transition probabilities depend on the action of the player he meets:

$$\begin{array}{cc} Q_Y(Y,\cdot,D)=q & Q_Y(Y,\cdot,H)=0 \\ Q_{Y_W}(Y,\cdot,D)=0 & Q_{Y_W}(Y,\cdot,H)=q \\ Q_{A_W}(Y,\cdot,D)=0 & Q_{A_W}(Y,\cdot,H)=1-q \\ Q_{A_S}(Y,\cdot,D)=1-q & Q_{A_S}(Y,\cdot,H)=0 \end{array}$$

- If a weak young individual meets an adult one, the transition probabilities only depend on its state:

$$\begin{array}{cc} Q_Y(Y_W,\cdot,D)=0 & Q_Y(Y_W,\cdot,H)=0 \\ Q_{Y_W}(Y_W,\cdot,D)=q & Q_{Y_W}(Y_W,\cdot,H)=q \\ Q_{A_W}(Y_W,\cdot,D)=1-q & Q_{A_W}(Y_W,\cdot,H)=1-q \\ Q_{A_S}(Y_W,\cdot,D)=0 & Q_{A_S}(Y_W,\cdot,H)=0 \end{array}$$

- Analogously, when an adult individual of type i meets another adult, the transition probabilities don't depend on the action of the opponent but only on the player state:

$$Q_{A_j}(A_i,\cdot,\cdot)=\begin{cases} p & i=j \\ 0 & i\neq j \end{cases}$$

$$Q_Y(A_i,\cdot,\cdot)=0 \quad Q_{Y_W}(A_i,\cdot,\cdot)=0$$

We define a mixed strategy u as the probability of playing H conditioned on the fact of being a strong adult:

$$u := \mathbb{P}(H|A_S) \qquad u \in (0,1) \tag{2}$$

The proportion of adults in the population corresponds to the expected lifetime spent in adult state over the total expected lifetime:

$$P(A)=\frac{\frac{1}{1-p}}{\frac{1}{1-p}+\frac{1}{1-q}} \tag{3}$$

We now suppose that the entire population is playing the common strategy v, except for one tagged player which plays u. The fraction of adults of type $i \in \{W,S\}$ in such a population is defined as:

$$\alpha_i(v) := \mathbb{P}(A_i|v)$$

This quantity only depends on the population's strategy v, not on the tagged player's one.

We have that the probability of being attacked when young is $\alpha_S(v)v$, which corresponds to the probability of finding a strong adult who plays aggressively. Consequently $\alpha_W(v) = P(A)(\alpha_S(v)v)$ and $\alpha_S(v) = P(A)(1-\alpha_S(v)v)$. We can explicit the fraction of strong and weak adults in the population:

$$
\begin{aligned}
\alpha_S(v) &= \frac{P(A)}{(1+P(A)v)} \\
\alpha_W(v) &= \frac{vP(A)^2}{(1+P(A)v)}
\end{aligned}
\tag{4}
$$

As in classical evolutionary game theory, we consider a fitness function related to the rate of reproduction of an individual. Only adults can reproduce, so we define immediate reward only for adult players.

The immediate payoff matrix, describing the fitness of the row player when meeting the column player, is the following:

	$A_S(H)$	$A_S(D)$	A_W	Y	Y_W
$A_S(H)$	$\frac{1}{2}-\delta$	1	1	1	1
$A_S(D)$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
A_W	$-\Delta$	$\frac{1}{2}-\Delta$	$\frac{1}{2}-\Delta$	$\frac{1}{2}-\Delta$	$\frac{1}{2}-\Delta$

where $\delta, \Delta > \frac{1}{2}$. The parameter δ represent the cost of the fight, whereas Δ reflects the negative fitness for individuals who had been attacked when young.

We define the expected immediate utility for an adult of type $i \in \{S, W\}$, playing u against a population playing v:

$$
R(i,u,v) := E_{u,v}[r(A_i, a, s', a')] \tag{5}
$$

where $s' \in \mathcal{S}$, $a \in \mathcal{A}$, $a' \in \mathcal{A}'$ (note that only when an individual is a strong adult it can choose an action) and $r(A_i, a, s', a')$ is the immediate reward of an individual in state A_i playing a against an individual in state s' playing a'. Since each player is born in young state, (5) doesn't depend on the initial state.

3 Deriving the Expected Fitness and the Equilibria

3.1 Computing the Expected Fitness

The expected fitness during the lifetime of an individual playing strategy u against a population playing v is given by:

$$
F(u,v) = \frac{1}{1-p}\left(\alpha_S(v)R(S,u,v) + \alpha_W(v)R(W,v)\right) \tag{6}
$$

where $R(S,u,v)$ ($R(W,v)$) is the expected immediate reward of an adult of type S (resp W) playing u against a population playing v. The expected immediate reward of a weak adult doesn't depend on the strategy u, because it has no choice of action.

$$
\begin{aligned}
R(S,u,v) = \Bigg[& u\left((1-P(A)) + \alpha_S(v)(1-(\frac{1}{2}+\delta)v) + \alpha_W(v)\right) + \\
& +(1-u)\left(\frac{1-P(A)}{2} + \frac{1-v}{2}\alpha_S(v) + \frac{\alpha_W(v)}{2}\right)\Bigg]
\end{aligned}
\tag{7}
$$

$$R(W,v)=\left[(1-P(A))(\frac{1}{2}-\Delta)-\alpha_S(v)(\frac{1}{2}-\Delta-\frac{v}{2})+(\frac{1}{2}-\Delta)\alpha_W(v)\right] \tag{8}$$

By substituting equations (7) and (8) in (6) we obtain:

$$\begin{aligned}F(u,v)=&\frac{\alpha_S(v)}{1-p}\left[u\left((1-P(A))+\alpha_S(v)(1-(\frac{1}{2}+\delta)v)+\alpha_W(v)\right)+\right.\\&\left.+(1-u)\left(\frac{1-P(A)}{2}+\frac{1-v}{2}\alpha_S(v)+\frac{\alpha_W(v)}{2}\right)\right]+\\&+\frac{\alpha_W(v)}{1-p}\left[(1-P(A))(\frac{1}{2}-\Delta)-\alpha_S(v)(\frac{1}{2}-\Delta-\frac{v}{2})+(\frac{1}{2}-\Delta)\alpha_W(v)\right]\end{aligned} \tag{9}$$

where $\alpha_S(v)=\frac{P(A)}{1+P(A)v}$ and $\alpha_W(v)=\frac{P(A)^2v}{1+P(A)v}$. By definition we have that $\alpha_S(v)+\alpha_W(v)=P(A)$; we thus obtain:

$$\begin{aligned}F(u,v)=&\frac{\alpha_S(v)}{1-p}\left[u\left(1-P(A)+P(A)-\alpha_S(v)(\frac{1}{2}+\delta)v\right)+\right.\\&\left.+(1-u)\left(\frac{1-P(A)}{2}+\frac{P(A)}{2}-\frac{v\alpha_S(v)}{2}\right)\right]+\\&+\frac{\alpha_W(v)}{1-p}\left[\frac{1}{2}-\Delta-\alpha_S(v)\frac{v}{2}\right]=\\&=\frac{1}{1-p}\left[\alpha_S(v)\left(\frac{1}{2}-\delta v\alpha_S(v)\right)u-\Delta\alpha_W(v)+P(A)\left(\frac{1}{2}-\frac{v}{2}\alpha_S(v)\right)\right]\end{aligned} \tag{10}$$

3.2 Computing and Classifying Equilibria

We first look for the equilibrium in pure strategies. If the population's strategy is $v=0$, that is if everyone is not aggressive, we obtain:

$$F(u,0)=\frac{P(A)(u+1)}{2(1-p)} \tag{11}$$

This implies that strategy $u=0$ is never an equilibrium. We consider the case $v=1$. The utility function equals:

$$F(u,1)=\frac{P(A)\left[u+uP(A)-2v\delta+1+P(A)-2\Delta P(A)-2P(A)^2\Delta\right]}{2(1-p)(1+P(A))^2} \tag{12}$$

By substituting $u=0$ and $u=1$ we have:

$$F(0,1)=-\frac{P(A)(2\Delta P(A)-1)}{2(1+P(A))(1-p)}$$

$$F(1,1)=\frac{P(A)(1+P(A)-\delta P(A)-\Delta P(A)-\Delta P(A)^2)}{(1-p)(1+P(A))^2}$$

In order for $u=1$ to be a pure equilibrium the inequality $F(1,1)\geq F(0,1)$ must hold. We have that:

$$F(1,1) - F(0,1) \geq 0 \quad \Leftrightarrow -\frac{P(A)(2\delta P(A) - 1 - P(A))}{2(1+P(A))^2(1-p)} \geq 0$$

This is true if and only if

$$1 + P(A) - 2\delta P(A) \geq 0$$

We thus obtain that the strategy $u = 1$ is an equilibrium in pure strategies if and only if:

$$\delta \leq \frac{1 + P(A)}{2P(A)} \tag{13}$$

We now look for the equilibrium in mixed strategies: we apply the indifference principle, which states that an equilibrium is a mixed one only if the player is indifferent between its pure strategies. We thus impose the indifference among the two possible pure strategies of the first player: $u = 0$ and $u = 1$.
By substituting $u = 0$ in (10) we obtain:

$$\begin{aligned} F(0,v) &= \frac{1}{1-p}\left[P(A)\left(\frac{1}{2} - \frac{v\alpha_S(v)}{2}\right) - \Delta\alpha_W(v)\right] = \\ &= \frac{1}{1-p}\left[\frac{P(A)(1 - 2\Delta P(A)v)}{2(1+P(A)v)}\right] \end{aligned} \tag{14}$$

For $u = 1$ (10) becomes:

$$F(1,v) = \frac{1}{1-p}\left[\alpha_S(v)\left(\frac{1}{2} - \delta v\alpha_S(v)\right) - \Delta\alpha_W(v) + P(A)\left(\frac{1}{2} - \frac{v}{2}\alpha_S(v)\right)\right] \tag{15}$$

For v to be a mixed equilibrium, the following equation has to hold: $F(0,v) = F(1,v)$. By solving it, we find the following value:

$$v_* = \frac{1}{P(A)(2\delta - 1)} \tag{16}$$

In order for v_* to be a mixed strategy, it has to satisfy $0 < v_* < 1$; by imposing these constraints on (16) we obtain the following condition:

$$\delta \geq \frac{1 + P(A)}{2P(A)}$$

We define the threshold

$$\delta_* = \frac{1 + P(A)}{2P(A)}$$

and we summarize the results we obtained in the following theorem.

Theorem 1. *Given the game described in Section 2.2 we have that:*

1. *if $\delta < \delta_*$ the unique equilibrium strategy of the game is the pure strategy $u = 1$.*

2. *if $\delta \geq \delta_*$ the unique equilibrium strategy of the game is the mixed strategy $v_* = \frac{1}{P(A)(2\delta-1)}$*

In figure 1 we plot the mixed equilibrium strategy v_* obtained in (16) as a function of the proportion $P(A)$ of adults in the population for three different values of δ: the continuous higher line is obtained with $\delta = 4$, the plotted line with $\delta = 6$, the continuous lower one with $\delta = 10$. As expected we can observe that v_* is a decreasing function of $P(A)$: this means that the higher the proportion of adults in the population is, the lower the value of the equilibrium is, that is the lower the probability of being aggressive is.

In figure 2 the mixed equilibrium strategy v_* is plotted as a function of the threshold δ_*; the continuous higher line is obtained with $P(A) = 0.15$, the dotted line with $P(A) = 0.5$, the continuous lower one with $P(A) = 0.88$. As the value of δ_* represents the cost of the fight between two aggressive adults, as expected, we can see that v_* is decreasing in δ_*.

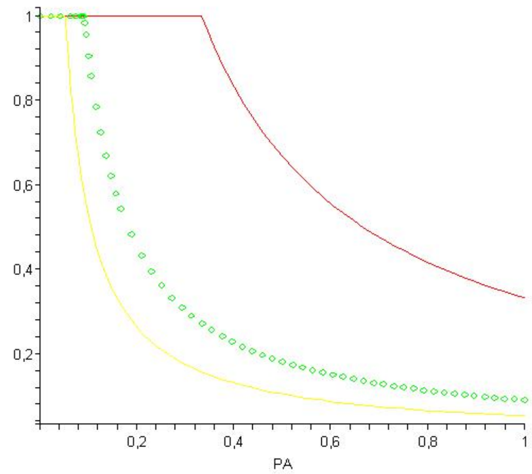

Figure 1: The equilibrium strategy v_* as a function of the proportion $P(A)$ of adults in the population for different values of δ. The continuous higher line is obtained with $\delta = 4$, the dotted line with $\delta = 6$, the continuous lower one with $\delta = 10$.

Remark 1. *We observe that the equilibrium value v_* does not coincide with the standard Hawk and Dove game's Nash equilibrium $x = \frac{1}{2\delta}$ even when $p \to 1$ (so that $P(A) \simeq 1$) because we may have at equilibrium*

$$\liminf_{p \to 1} \alpha_W^* > 0$$

As a matter of facts, even if there are almost only adult individuals in the population, the fraction of weak adults may not be negligible.

4 Group players

4.1 Reformulating the Hawk and Dove Game as a GMDEG problem

In this section we distinguish among actors, that is the individuals which interact, and players, that is the **groups of individuals whose utility is maximized**, in the framework of MDEG. The rules that determines the strategy of individuals are chosen as to maximize the utility of their group.

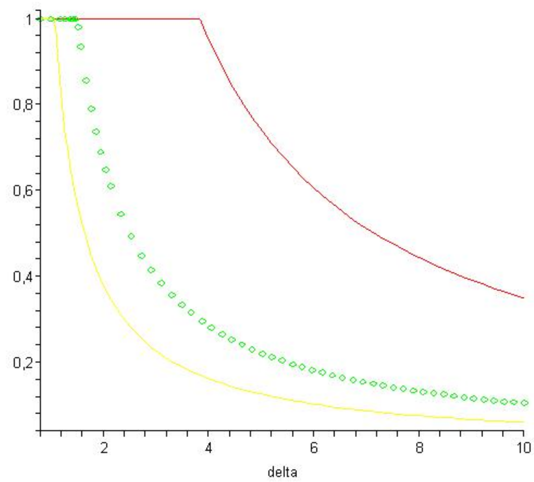

Figure 2: The equilibrium strategy v_* as a function of the value of the threshold δ_* for different values of $P(A)$. The continuous higher line is obtained with $P(A) = 0.15$, the dotted line with $P(A) = 0.5$, the continuous lower one with $P(A) = 0.88$.

We suppose that the infinite population is divided into N symmetric groups of players. Each group is characterized by its level of aggressiveness:

$$u_i := \mathbb{P}(H|A_s^i) \qquad i = 1, \ldots, N \tag{17}$$

We define the vector of groups' strategies:

$$\bar{u} = (u_1, \ldots, u_N)$$

The probability of being a strong or a weak adult doesn't depend on the group the player belongs to; they are defined as:

$$\alpha_S(\bar{u}) := \mathbb{P}(A_S|\bar{u})$$
$$\alpha_W(\bar{u}) := \mathbb{P}(A_W|\bar{u})$$

We assume that $\alpha_S(\bar{u})$ and $\alpha_W(\bar{u})$ depend on the average quantity $\underline{u} := \frac{\sum_{l=1}^N u_l}{N}$, that is the average probability of being aggressive in the population:

$$\alpha_S\left(\frac{\sum_{l=1}^N u_l}{N}\right) = P(A)\,(1 - \alpha_S(\underline{u})\underline{u})$$
$$\alpha_W\left(\frac{\sum_{l=1}^N u_l}{N}\right) = P(A)\,(\alpha_S(\underline{u})\underline{u})$$

We are thus supposing that, when deciding which action to take, an individual does not distinguish whether it interacts with someone from its own group or not. We can explicit the previous equations:

$$\alpha_S(\underline{u}) = \frac{P(A)}{1 + P(A)\underline{u}} \tag{18}$$

$$\alpha_W(\underline{u}) = \frac{\underline{u}P(A)^2}{1+P(A)\underline{u}} \tag{19}$$

When the entire population plays the same strategy u, except for a fixed group i playing u_i we have that:

$$\underline{u} = \frac{u_i + (N-1)u}{N}$$

4.2 Group Fitness

We consider the same payoff matrix defined in Section 2.2. The expected fitness during the lifetime of a player i, that is of group i, in a population playing u is given by:

$$\begin{aligned} F(u_i, u) = \frac{1}{1-p} \Bigg[& \alpha_S(\underline{u}) \left(\frac{1}{N} R(S, u_i, u_i) + \frac{N-1}{N} R(S, u_i, u) \right) \\ & + \alpha_W(\underline{u}) \left(\frac{1}{N} R(W, \cdot, u_i) + \frac{N-1}{N} R(W, \cdot, u) \right) \Bigg] \end{aligned} \tag{20}$$

where:

$$\begin{aligned} R(S, u_i, u) &= u_i \left((1-P(A)) + \alpha_S(\underline{u})(1 - (\frac{1}{2} + \delta)u) + \alpha_W(\underline{u}) \right) \\ &+ (1-u) \left(\frac{1-P(A)}{2} + \frac{1-v}{2}\alpha_S(\underline{u}) + \frac{\alpha_W(\underline{u})}{2} \right) \\ &= u_i \left(\frac{1-P(A)}{2} + \alpha_S(\underline{u})(\frac{1}{2} + \delta)u) + \alpha_W(\underline{u}) \right) \\ &+ \left(\frac{1-P(A)}{2} + \frac{1-v}{2}\alpha_S(\underline{u}) + \frac{\alpha_W(\underline{u})}{2} \right) \end{aligned}$$

is the expected immediate reward of a strong adult in group i, playing strategy u_i against a population playing u and

$$R(W, \cdot, u) = (1-P(A))(\frac{1}{2} - \Delta) - \alpha_S(\underline{u})(\frac{1}{2} - \Delta - \frac{u}{2}) + (\frac{1}{2} - \Delta)\alpha_W(\underline{u})$$

is the expected immediate reward of a weak adult in a population playing u. Analogously for $R(S, u_i, u_i)$ and $R(W, \cdot, u_i)$. $\alpha_S(\underline{u})$ and $\alpha_S(\underline{u})$ are defined in (18) and (19) with $\underline{u} = \frac{u_i + (N-1)u}{N}$ and $\frac{1}{N}$ and $\frac{N-1}{N}$ are respectively the probability for a player of meeting an opponent in the same group and in a different group.

4.3 Equilibria

In order to obtain insight on the impact of the groups on the equilibrium, we shall focus on some particular values of the parameters. From now on, we will consider the case of two groups, i.e. we set $N = 2$.

We first look for equilibria in pure strategies. We suppose that the group players are not aggressive, i.e. we substitute $u = 0$ in (20) and we compare the results obtained for the two possible response of the tagged group i, $u_i = 0$ and $u_i = 1$.

$$F(0,0) = \frac{-1}{2P(A)(p-1)}$$

$$F(1,0) = \frac{-P(A)(4 + 2P(A) - 2P(A)\delta - P(A)^2 - 2P(A)\Delta + 3P(A)^2\Delta + P(A)^3\Delta - P(A)^3\delta)}{((p-1)(2+P(A))^2)}$$

In order to find a pure equilibrium, we have to estimate the difference:

$$F(0,0) - F(1,0) = -\frac{P(A)(-4 + 3P(A)^2 + 4P(A)\delta + 4P(A)\Delta - 6P(A)^2\Delta - 2P(A)^3\Delta + 2P(A)^3\delta)}{2((-1+p)(2+P(A))^2)} \tag{21}$$

If the population is aggressive, i.e. if $u = 1$, we have that:

$$F(0,1) = \frac{P(A)(-2 - P(A) + P(A)^2 + 2P(A)\Delta - 3P(A)^2\Delta - P(A)^3\Delta + P(A)^3\delta)}{(-1+p)(2+P(A))^2}$$

$$F(1,1) = \frac{P(A)(-1 - P(A) + P(A)\delta + P(A)\Delta - P(A)^3\Delta - P(A)^2\Delta + P(A)^3\delta)}{(-1+p)(1+A)^2}$$

The difference equals:

$$\begin{aligned} F(1,1) - F(0,1) = \\ \frac{P(A)(-2 - 3P(A) - 2P(A)^2 + 4P(A)\delta + 2P(A)\Delta - A^2\Delta}{((-1+p)(1+P(A))^2(2+P(A))^2)} + \\ \frac{+4P(A)^2\delta - 2P(A)^3\Delta + 4P(A)^3\delta - 2P(A)^3 + 2P(A)^4\delta - P(A)^4)}{((-1+p)(1+P(A))^2(2+P(A))^2)} \end{aligned} \tag{22}$$

By fixing the value of $\Delta = 1$ and of the fraction of adults in the population $P(A) = 0.5$ we are able to compute the differences in (21) and in (22).
In the case of a non aggressive population we obtain:

$$F(0,0) - F(1,0) = -\frac{3(-4+3\delta)}{100(p-1)} > 0$$

$$\Leftrightarrow \delta > \frac{4}{3} = 1.\bar{3}$$

The result we found show the emergence of a new interesting phenomenon: when considering group players, we have that for the values of the parameters we have fixed, there exists a threshold $\underline{\delta}$ such that $\forall \delta > \underline{\delta}$ the pure strategy $u = 0$ is an equilibrium.

When the population is aggressive, we obtain:

$$F(1,1) - F(0,1) = \frac{(-61+58\delta)}{450(p-1)} > 0$$

$$\Leftrightarrow \delta < \frac{61}{58} \simeq 1.05$$

Theorem 2. *For $N = 2$, $P(A) = 0.5$ and $\Delta = 1$, there exist $\underline{\delta}, \bar{\delta}$ such that the pure strategy equilibrium of the group game is:*

- $u_* = 0 \ \forall \delta > \underline{\delta}$;
- $u_* = 1 \ \forall \delta < \bar{\delta}$.

where $\underline{\delta} = 1.\bar{3}$ *and* $\bar{\delta} = 1.05$

We now seek for Nash equilibria in mixed strategies. By substituting $N = 2, P(A) = 1/2, \Delta = 1$ in (20) we get:

$$F(u_i, u) = \frac{(-16u_i - 5u_i^2 - 6u_i u + 9\delta u_i^2 - 16 + 10u_i\delta u - u^2 + \delta u^2)}{4((-1+p)(4+u_i+u)^2)}$$

We maximize the utility function:

$$\frac{\partial F(u_i, u)}{\partial u_i} = \frac{(-8 - 6u_i - 10u - u_i u - u^2 + 18\delta u_i + 2u_i\delta u + 10\delta u + 2\delta u^2)}{((-1+p)(4+u_i+u)^3)}$$

By imposing:

$$\frac{\partial F(u_i, u)}{\partial u_i} = 0$$

we find:

$$u_i = \frac{-8 - 10u - u^2 + 10\delta u + 2\delta u^2}{-6 - u + 18\delta + 2\delta u} \tag{23}$$

We now impose the symmetry $u_i = u$ and we obtain the two solutions:

$$u^1 := -\frac{-4 + 7\delta - \sqrt{12 - 48\delta + 49\delta^2}}{(-1 + 2\delta)}$$

$$u^2 := \frac{-(-4 + 7\delta + \sqrt{12 - 48\delta + 49\delta^2}}{(-1 + 2\delta)}$$

The second solution u^2 is always negative and thus it is not acceptable. In figure 4.3 we plot the value of u^1 as a function of δ, where $P(A) = \frac{1}{2}$ and $\Delta = 1$.

If we consider the results obtained in pure strategies, we can observe another new interesting phenomenon: we can determine two intervals in which we have two equilibria: a mixed and a pure ones.

More precisely, we have that: :

Theorem 3. *Given the model described in Section 4, for the fixed values of the parameters $N = 2$, $P(A) = 0.5$, $\Delta = 1$, the Nash equilibria of the groups game are*

1. *for $0.5 < \delta < 0.8125$ one equilibrium in pure strategies $u_* = 1$;*
2. *for $0.8125 \leq \delta \leq 1,05$ two equilibria: the pure equilibrium $u_* = 1$ and the mixed equilibrium given by u^1 ;*
3. *for $1,05 < \delta < 1.\bar{3}$ one equilibrium in mixed strategies given by u_1;*
4. *for $1.\bar{3} \leq \delta$ two equilibria: the pure equilibrium $u_* = 0$ and the mixed equilibrium given by u^1;*

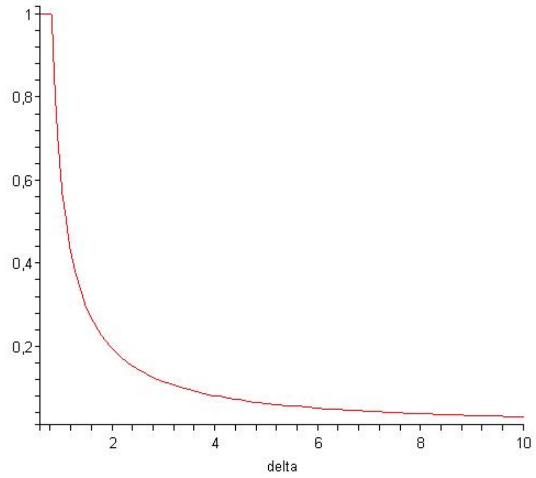

Figure 3: The value of the equilibrium in mixed strategies u^1 as a function of the value of δ

5 Discussion

We have revisited in this paper the Hawk and Dove game within the MDEG framework. In this context an individual may be in one of several states, and when meeting other individuals, the actions of both individuals involved in the interaction determine not only the immediate fitness but also the transition probabilities of the players' individual state. We found that the described game has only one equilibrium, which can be the pure aggressive strategy or a mixed equilibrium, depending on the value of the parameter relating to the cost of the fight δ.

A second important extension of the Hawk and Dove game consisted in considering a whole group of individuals as corresponding to one player. We thus depart from the basic concept of Evolutionary Game Theory in which each individual is a player. This is motivated by observing that in many cases, reproduction within a species is not done by the individuals that interact with each other and hence fitness in the Darwinian sense cannot be attributed to these but to the entire group. We assumed here that individuals are not aware of the group that their competitor belongs to. A strong individual may thus harm a young individual of his own group if he behaves aggressively.

We saw that the equilibrium in this game brings important novel features with respect to the classical game: for fixed values of the parameters of the game we found that the non aggressive strategy can be an equilibrium in pure strategies, which is never the case for the Hawk and Dove problem in MDEG. We also determined some specific intervals of values of the parameter δ for which the game has two equilibria, a mixed equilibrium and a pure one.

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References

[1] E. Altman, R. El-Azouzi, Y. Hayel, and H. Tembine. An evolutionary game approach for the design of congestion control protocols in wireless networks. In *Physicomnet workshop, Berlin, April 4*, 2008.

[2] Eitan Altman, Rachidh El-Azouzi, Yezekael Hayel, and Hamidou Tembine. The evolution of transport protocols: An evolutionary game perspective. *Computer Networks*, 53:1751–1759, July 2009.

[3] Eitan Altman and Yezekael Hayel. Markov decision evolutionary games. *IEEE Transactions on Automatic Control*, 55(6), June 2010.

[4] J. Maynard Smith. Game theory and the evolution of fighting. *J. Maynard Smith, On Evolution (Edinburgh: Edinburgh University Press)*, pages 8–28, 1972.

[5] J. Maynard Smith. *Evolution and the theory of Games.* Cambridge University Press, UK, 1982.

[6] Hamidou Tembine, Eitan Altman, Rachid El Azouzi, and Yezekael Hayel. Evolutionary games in wireless networks. *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, 40(3):634–646, 2010.